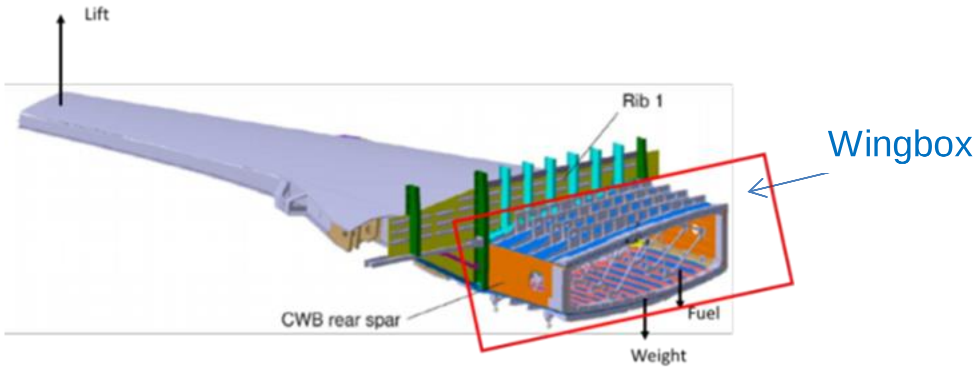
AIRBUS QUANTUM COMPUTING CHALLENGE

Problem Statement n°4

WINGBOX DESIGN OPTIMISATION

Multi-Disciplinary Challenge: Wingbox Design Optimisation



Multidisciplinary design optimization is an ongoing challenge in the aerospace industry, resulting in long design lead times and untapped optimisation potential. Quantum computing may offer a viable path towards efficient multi-parameter optimization covering the entire design space. Here we ask for the application of quantum computing solutions to a problem involving airframe loads, mass modelling and structural analysis. The target is to preserve structural integrity while optimising weight. Weight optimisation is key to low operating costs and reduced environmental impact. The challenge arises when computing a broad range of aircraft design configurations simultaneously which is currently not possible with classical computing.

Structural integrity is demonstrated by simulating key flight occurrences required by air worthiness regulations. A representative case is selected and presented as the challenge in a simplified form. A model of an aircraft is exposed to a static (time-independent) manoeuvre loading or to a dynamic gust load (time-dependent), under a variety of fuel distributions and in a variety of flight conditions. Structural sizing parameters of the wingbox shall be optimised to obtain a minimum weight solution.

In essence, in the simplest case, we are looking for a vector p of structural parameters, such that a linear functional w(p) that corresponds to the mass is minimized, while the following constraint is satisfied: the set of linear systems $K(p)\langle x\rangle = F^{\,j}$ for a fixed matrix K parametrized by p and vectors F^j, for j in {1,L}, have a solution for which $RF(\langle x\rangle) > 1$, for a given reserve function RF. In the technical dossier, more complex cases are described.

Note that displacements $\langle x\rangle$ are transformed to internal loads which can depend on parameters. For this reason it is preferable to work with stress constraints based on stress allowables where RF is a ratio between a constraint and a minimum (negative) or maximum value. Possibly Von Mises limits can also be used.

Requested Results

1. *Find the lowest structural weight solution across the entire design space*
   How? Computation of reserve factors for wing for a set of fuel / flight condition scenarios using a combination of quantum and classical computing algorithms
   KPI → assess accuracy of the solution against classical computing
2. *Assess quantum computing capability to scale in performance when problem is made larger or more complex*

KPI ➔ assess computation speed against complexity (ex: increasing number of equations, interaction of different equations, dependencies)
KPI ➔ assess computation speed against scalability (increasing number of parameters, simultaneous simulations)

Technical Description

The technical description is a list of hypotheses that allows to create a viable problem yet restricting as much as possible the number of variables to be programmable on quantum computers. However increasing complexity is also proposed to experiment quantum computing scalability for these problems.

- *Geometrical and Structural boundary conditions:*
  - *Wing shape is fixed, Beam shape is given, with front spar and rear spar, upper and lower wing cover, n ribs*
  - *5 structural parameters for wingbox:*
    *→ thickness of upper wing cover : **p₁** = t_u*
    *→ thickness of lower wing cover : **p₂** = t_l*
    *→ thickness of spars : **p₃** = t_{sp}*
    *→ thickness of ribs : **p₄** = t_r*
    *→ area of stiffeners : **p₅** = A_{st}*
    *→ 2 choices of material, "**p₆**"*
    ➔ two different young modulus E_{mat} and density ρ_{mat}
- *Mass properties*
  - *Structural wing weight to be derived from design and material properties*
  - *Engine mass fixed, Fuel movement to be neglected*
  - *Fuel tank filling factor could be evaluated for different computation cases*
- *Loading of the wing:*
  - *Manoeuvre case (static) we assume static loadings to achieve 2.5g.*
  - *Vertical gust (dynamic), we consider gust time profile with different lengths L*
- *Structure analysis:*
  - *Structural integrity of the wing must be always ensured (Reserve Factors>1)*
    - *We retain only Von Mises Stress*

Target: Optimize **p** structural parameters (ex: "thickness of upper wing cover" - see above) to achieve the lightest overall **structural weight** for a loading computation on an aircraft model the constraint of reserve factors >1

Further Hypotheses

Mass is time independent (load computation)
We want to consider J cases of loading (Fuel, Mach, Gust length…)
Flight mechanics is considered linear when applicable.

Definition: **structural weight**.

Scalar function of **p**, $w(\boldsymbol{p})$
$w(\boldsymbol{p}) = W * M_s(\boldsymbol{p})$

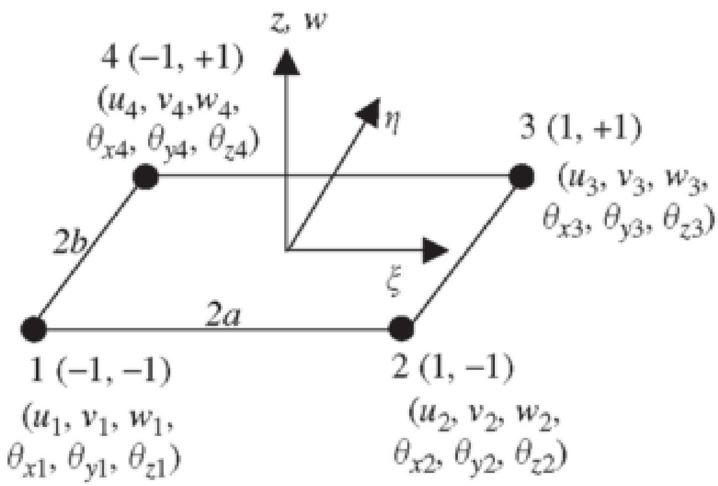
In our case this function will be a bilinear sum because the parameters chosen as structural are directly proportional to shell thickness and stiffeners area multiplied by the density of the parameter chosen. In the formula below all the parameters "p" have been made explicit. The structural weight is basically the volume of the pieces we want to optimize times the density of the chosen material.

$$w = \boldsymbol{\rho_{mat}}\left(A_u * \boldsymbol{t_u} + A_l * \boldsymbol{t_l} + A_{sp} * \boldsymbol{t_{sp}} + A_r * \boldsymbol{t_r} + l_r * \boldsymbol{A}_{st}\right)$$

Please consider that each of the parameters have in practical terms only some possible discrete values due to industrial constraint.

Definition: **structural displacements**.

$\langle x \rangle$: structural displacements – Nx1 vector representing the displacements of the nodes in finite element model. It represents all the possible degrees of freedom by which our structural model can move. As an example, a 4 nodes shell element has 24 degrees of freedom (12 displacements and 12 rotations). A structural model is made by connecting hundreds of these elements.



The vector of displacements for the single node "1" is in this case:

$$\langle x_1 \rangle = \left\langle \begin{matrix} u_1 \\ v_1 \\ w_1 \\ \theta_{x1} \\ \theta_{y1} \\ \theta_{z1} \end{matrix} \right\rangle$$

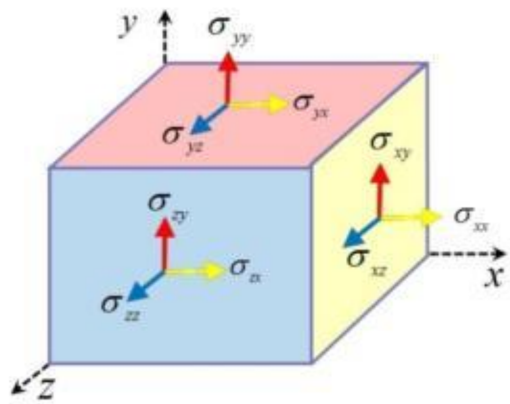
Definition: **Stiffness matrix**.

$K(\boldsymbol{p})$ is the stiffness matrix. It has N*N elements where N is the number of structural degrees of freedom. It expresses the structural stiffness linking these degrees of freedom (like the spring in a simple spring-mass-damper system), in other words it expresses the resistance of an element to deform against an applied force.

Definition: **Force vector**.

$F^{\,j}$. The force vector expresses how the structure is loaded.
The suffix "j" represents the fact that the wing structure can be loaded in j different manners, depending on the way the wing is filled with fuel, the aircraft payload and its flight conditions. A set of these loading conditions will be provided.

Definition: **Reserve factor**.



$RF(\langle x\rangle)$ is a function defining structural integrity. There are lots of functions for different failure types. Many of these criteria are based on stress.
The stress applied to a material is the force per unit area applied to the material. The maximum stress a material can stand before it breaks is called the breaking stress or ultimate tensile stress**.** For our case, stresses are a computed as a linear function of the displacements. Each element can have up to 6 stresses (less for 2D or 1D elements)

Each of these stresses can be computed as a linear combination of its displacements.

$$\langle\sigma\rangle = B(p)\langle x\rangle\langle x\rangle$$

B is a block diagonal matrix. A single, sufficiently simple function to compare the stress to maximum will be selected among those currently used. An example is the Von Mises Stress.

$$\sigma_{VM} = \sqrt{\frac{\left(\sigma_{xx}-\sigma_{yy}\right)^2+\left(\sigma_{yy}-\sigma_{zz}\right)^2+(\sigma_{xx}-\sigma_{zz})^2+6\left({\sigma_{xy}}^2+{\sigma_{yz}}^2+{\sigma_{xz}}^2\right)}{2}}$$

Once the Von Mises stress is computed it can be compared to a reference stress value for the material chosen σ_{MAX}

$$RF > 1 \quad \rightarrow \sigma_{VM} < \sigma_{MAX}$$

To preserve structural integrity this condition has to hold true for each element in the model.

Technical Problem Statement

In this framework we identify 4 classes of problems, of which only the first will constitute the target of the challenge (relevant data is under production).

1) Static Loading, without aeroelastic interaction, fixed flight mechanics

Ignoring flight mechanics and aeroelastic interactions corresponds to ignoring the effect that optimizing the structural weight may have on the loading of the wing via these neglected interactions. In this form the problem assumes the form of:

$$minimise\ w(\boldsymbol{p})\ under\ constraint\ of\ RF(\langle x\rangle) > 1\ with\ \langle x\rangle\ \ solution\ of\ K(p)\langle x\rangle = F^{\ j}$$

F^j is the static loading for the j-th case. In this case the problem is a clamped loaded wing problem. In this form the solution can be further expressed as

$$minimise\ w(\boldsymbol{p})\ \text{under constraints of}\ \ RF\left(K(p)^{-1}F^j\right) > 1$$

This means minimising a relatively simple functional of p under a large number of constraints (there are at N ~10^4 structural elements to check * j cases). And this considering only the previously described Von Mises reserve factor case (there are over 40 possible other criteria to choose and satisfy!).

It is interesting to notice that it is possible to considerably reduce the dimensionality of the deformation computation problem by using a modal analysis, which basically means considering only a linear combination of linearly independent possible deformations, called modes.

$$\langle x\rangle = U\ \langle \mu\rangle$$

U is a NxN matrix, but only a subset ("sel" for selection) of this matrix can be chosen (normally the modes having the lowest frequencies) to approximate the solution of the structural equation.

$$K(p)\langle x\rangle = F^{\ j} \quad \cong \quad k(p)\langle \mu_{sel}\rangle = f^{\,j}$$
$$\langle x\rangle \cong U_{sel}\ \langle \mu_{sel}\rangle\ , \qquad \langle F\rangle \cong U_{sel}\ \langle f\rangle\ , \qquad U_{sel}\ is\ a\ Nxn\ matrix, n \ll N$$

This allows to reduce considerably the matrix inversion step seen above.

Approximate Formulation: Explanation

Definition: **Mass matrix**.

$M(\boldsymbol{p})$ is the Mass matrix. It has N*N elements where N is the number of structural degrees of freedom (independent displacements of the nodes of our structural model). It expresses the inertia bound to these degrees of freedom (like the mass in a simple spring-mass-damper system). Mass matrix can be split into a structural mass (dependent on **p**) and a non_structural mass, dependent on the j-th filling case:

$$M^j(p) = M_s(\boldsymbol{p}) + M^j{}_{F+O}$$

Where subscript “s” stands for structural (the mass matrix of the structure we want to optimize) and “F+O” for fuel + other.
Please notice that it is not the same thing as the mass function w(p) described above as that is just sum of all the relevant masses for optimization. The mass matrix connects the mass and inertias to the single displacements. The mass matrix is introduced here as a concept, but is not used in the first proposed problem.

The matrix U provided before are found by solving the associated eigenvalue equation, that expresses the free vibration of the structural system

$$M(p)\langle\ddot{x}\rangle + K(p)\langle x\rangle = 0$$
$$\rightarrow M(p)\, U\, \lambda\ + K(p)U = 0$$

More Complex Problems

What follows are more complex formulations of this problem. They are proposed in case a quick and satisfying resolution of the first problem is provided. Data will be provided accordingly.

Definition: **Damping matrix**.

$C(\boldsymbol{p})$ is the damping matrix. It has N*N elements where N is the number of structural degrees of freedom (independent displacements). It expresses the structural damping linking these degrees of freedom (like the damper in a simple spring-mass-damper system). The damping matrix is introduced here as a concept, but is not used in the first proposed problem.

2) Dynamic Loading without aeroelastic interaction

$$minimise\ w(\boldsymbol{p})\ under\ constraint\ of\ RF(\langle x\rangle) > 1\ with\ \langle x\rangle\ \ solution\ of:$$

$$M^j(p)\langle\ddot{x}\rangle + C(p)\langle\dot{x}\rangle + K(p)\langle x\rangle = F(t,\langle x\rangle)^j$$

$F(t)^j$ is the dynamic loading for the j-th case. As difference from case 1, equations of motion of the aircraft cannot be excluded and must be included in the system.

3) Static Loading with aeroelastic interaction and trim

$$minimise\ w(\boldsymbol{p})\ under\ constraint\ of\ RF(\langle x\rangle) > 1\ with\ \langle x\rangle\ \ solution\ of\ K(p)\langle x\rangle = F(p,\langle x\rangle)^j$$

Same as 1) but: $F(\langle x\rangle)^j$ depends on the displacements as well. As difference from case 1, equations of motion of the aircraft cannot be excluded. This is not explicitly displayed here, but it corresponds to solve a further system of equilibrium equations.

4) Dynamic Loading with aeroelastic interaction

$$minimise\ w(\boldsymbol{p})\ under\ constraint\ of\ RF(\langle x\rangle) > 1\ with\ \langle x\rangle\ \ solution\ of:$$

$$M^j(p)\langle\ddot{x}\rangle + C(p)\langle\dot{x}\rangle + K(p)\langle x\rangle = F\big(p,t,\langle x\rangle,\langle\dot{x}\rangle\big)^j$$

$F\big(p,t,\langle x\rangle,\langle\dot{x}\rangle\big)^j$ is the dynamic loading for the j-th case. As difference from case 2, equations of motion of the aircraft cannot be excluded and must be included in the system. This is not explicitly displayed here, but it corresponds to solve a further system of equilibrium equations.

Additional Information and Input Files

Sample calculation and input files to support the mathematical formulation mentioned above are available for download.
Data file: calculation_example.m / data_delivery_2.docx
Script file to load on MATLAB: data_problem.mat

The Situation in Airbus Today

This type of problems is currently faced in Airbus in a multistep approach. The structural model is initially reduced to a simpler structure, on which many loading cases are applied. The loading cases resulting more critical to this simplified structure are then used for detailed design, in which automatic optimization is only partially applied. The dynamic cases are often solved using a modal approach and using Laplace/Fourier transforms (but not exclusively). Once the most critical loading cases are highlighted these are used for detailed design.