SYSTEMA PERTURBATIONS

Version 4.9.3

User Manual

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All suggestion or error concerning the software or this software manual can be sent to:

Airbus Defence and Space

For the attention of Ms C. BAYEUX

Z.I. du Palays

31 rue des Cosmonautes

31402 TOULOUSE CEDEX 4

FRANCE

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Introduction

During normal flight conditions, planet orbiting spacecraft are subjected to a variety of perturbing forces produced by the influence of the Sun and the planets. In order to evaluate the effects on the trajectory and attitude, it is necessary to model the interaction of the spacecraft with its environment.

The Earth atmosphere is the major contributor of satellite perturbations at very low altitudes. The atmosphere is also relatively poor known due to the lack of data, the complexity of atmospheric dynamics and the interaction with the solar wind, only known through statistical prediction tables. This perturbation is computed by the software under the term **Air Drag**. The software is also able to compute force, torque and thermal flux on a spacecraft in orbit around another planet thanks to a user defined atmosphere model.

The Solar radiation pressure is the major perturbation caused by light fluxes incident on the spacecraft. This perturbation is computed by the software under the term **Solar radiation pressure**.

1 Software Overview

1.1 Satellite model, orbit and attitude definition

The Modeler tab of the software is dedicated to the definition of a geometrical and physical model of the spacecraft. The Trajectory, Kinematics and Mission tabs allow to describe the orbit, the pointing and the kinematics of the satellite. In the Mission tab you can also define the computation points that correspond to the orbital position at which the perturbing forces will be computed by the software.

1.2 Calculations

In the Processing tab, you will use the Perturbations modules (Solar Pressure and Air Drag) to compute the forces, torques and fluxes along one or several orbits. The Perturbations application calculates, at each orbital position and separately, the solar radiation pressure and/or the air drag in the satellite frame. It is possible to activate or deactivate any of these two computations.

1.3 Environmental Models

The environmental models for the skin-forces phenomena are the following:

- **Solar radiation:** the solar flux is given by the solar constant (W/m2). The solar constant can be either defined manually by the user (as a constant) or computed automatically by the software at each orbital position depending on the sun distance.

- **Air drag:** the Earth environmental model for the particles characteristics determination is NRLMSISE-00. It extends from the ground to the exobase (0 - 1000 km) and describes the average observed behaviour of temperature, 8 species densities, and mass density via a parametric analytic formulation. The NRLMSISE-00 model is associated to the HWM07 wind model which provides a statistical representation of the horizontal wind fields of the Earth's atmosphere from the ground to the exosphere (0 - 500 km). HWM07 may possibly be switched off in order to simplify the forces analyses. A simplified user defined atmosphere model is also available to allow calculations around other planets where Earth's models are not applicable. This user defined model is based on constant values for density, temperature and average molar mass.

1.4 Particle/surface interaction

For particles of the atmosphere and photons, absorption, reflection and transmission are considered. Reflection and transmission can be specular and diffuse. In addition, an accommodation coefficient defines, for particles, the proportion of particles which are reflected in a diffuse way at wall temperature.

1.5 Monte-Carlo Ray-Tracing technique overview

The Monte-Carlo Ray-Tracing technique is used for the forces torques and fluxes computation (radiation pressure, air drag).

The Monte-Carlo Ray-Tracing technique employs the statistical procedures for random processes to determine the average effects of incident flux on a satellite, by tracing the paths of a representative sample of individual rays or particles, and studying the effects of their interaction with the satellite. The use of this technique for the modelling of non-gravitational perturbation forces presents at least three advantages:

1) The technique is very well adapted to the modelling of surface impingement mechanisms. Furthermore, multiple reflection, shadowing, thermal agitation (of particles) are easily accounted for.

2) The complexity of the geometrical model does not necessarily make the cpu time prohibitive.

3) The technique handles shading effects very well.

The Monte-Carlo Ray-Tracing method contains the following modelling tasks:

- Emission of rays from a node

    o Emission points are sampled randomly, according to an iso-distribution law, with an analytic definition of shapes.

    o All the incident rays come from the same direction (sun direction for Solar Pressure or velocity direction for Air drag)

- Propagation of rays and search for impacts

    o The main problem is to determine which surface is stuck by the ray

    o An optimized search is needed as it is impossible to test all the surfaces

    o The Ray-Tracing considers the analytic definition of shapes

- Energy/flux transfers to impacted nodes

    o The ray has started from the initial emitting node with an energy E_0 or a particle flux N_0

    o During its travel, the ray transfers energy/flux to each impacted node

    o The ray is re-emitted from the impacted surface if the energy or the particle flux is greater than extinction threshold

- Re-emission of rays from impacted nodes

    o Re-emission direction determined by the Monte-Carlo method:

        ▪ Random choice between:

            • Diffuse reflection

            • Specular reflection

- Diffuse transmission

- Direct transmission

▪ This choice is consistent with the properties of the surface (absorptivity, specularity …) and with the analytic definition of shapes surfaces.

It is worth mentioning that the convergence accuracy of this method depends on the number of emitted rays. This number is a parameter of the software that can be set by the user. The convergence accuracy depends also on other parameters like the properties of the surfaces. For example with the same number of emitted rays, the accuracy will be lower on a diffuse surface than on a specular surface. It is therefore strongly recommended to check the convergence of the simulation and adapt the number of emitted rays to each case of calculation.

2 How to make a Perturbations simulation

The present section is intended to show the general organization of the Perturbations simulation.

The main objects of perturbations is the evaluation of perturbing forces and torques on planet orbiting spacecraft for AOCS studies. The perturbations considered in this software are: the sun radiation pressure and the air drag caused by the earth atmosphere (or user defined atmosphere on other planets).

The entire procedure of a perturbations analysis can be divided into five steps:

1) Space vehicle model generation

2) Spacecraft on orbit description

3) Simulation definition

4) Forces and torques computation

5) Results Analysis

The five steps will be described in the following paragraphs. Details on parameters for tuning and analyzing a simulation are given in sections 3 and 4.

2.1 Geometrical and physical model

The first step in the perturbations analysis is the space vehicle model generation in the Modeler tab of Systema. The Modeler tab is used to work on models and meshing. The elements of the geometrical model have physical properties used for Perturbation analyses.

The geometrical model can be imported thanks to the File/Open/model menu. The menu lists the type of files recognized in the user's working directory that can possibly be input as geometrical model. The type of files is recognized through the file extension and can be selected from the drop-down menu. It is also possible to create a new geometrical model (File/New) which will be saved as a ".sysmdl" file. For more information about the geometrical model management, please refer to the Systema User Manual ("Geometrical Model Management" chapter).

Once the geometrical model is created, it is possible to assign coating properties to each surfaces. These properties can be consulted or modified using the edition window, via a specific tab called "Perturbations".

The properties can be in "Inherited configuration" or in "Local configuration". The local configuration allows modifying the "Side management" of the container which may propose to set alike properties for both sides or distinct properties for the positive and negative sides.
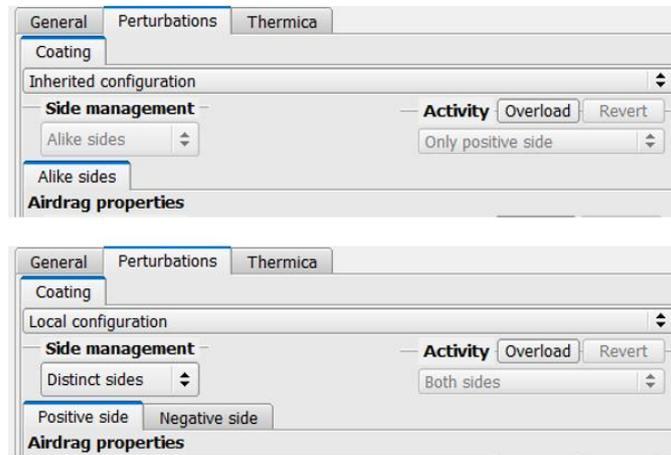
Figure 2.1-1 Alike or distinct side management

When a particle of the atmosphere (Air Drag only) strikes a surface, three phenomena may occur:

- **Absorption:** the particle/photon sticks to the surface. For instance consider deposits or chemical reactions of particles on sensitive surfaces.
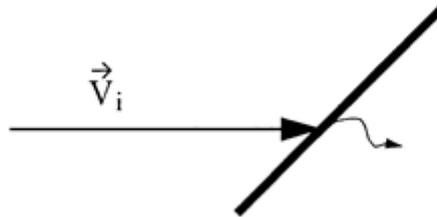
Figure 2.1-2 Absorption

- **Reflection:** The particle rebounds on the surface, either specularly or diffusely. When specular, the normal component of the velocity direction is inverted whereas the tangential component is conserved.
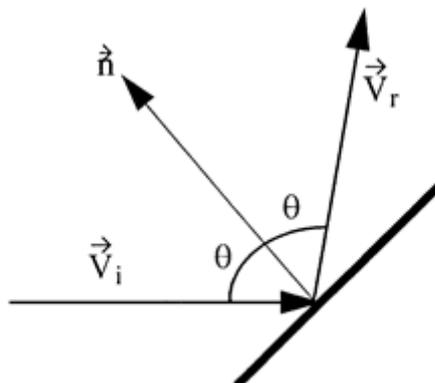
Figure 2.1-3 Specular reflection

Two cases of diffuse reflection can be considered:

- o the particle keeps the same velocity magnitude
- o the particle is re-emitted at a velocity function of the surface temperature. This possibility is modelled by the accommodation coefficient.
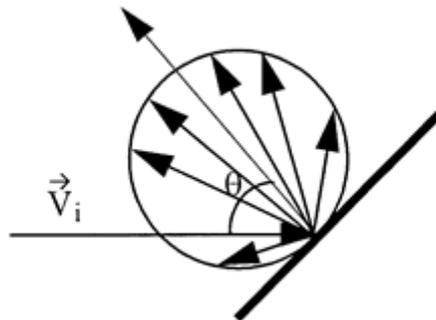
Figure 2.1-4 Diffuse reflection

- **Transmission:** the particle passes through the surface with the same velocity vector (direct transmission) or with random velocity vector following the cosine law (diffuse transmission). The transmission is useful to model grid surfaces (antennas, trusses).
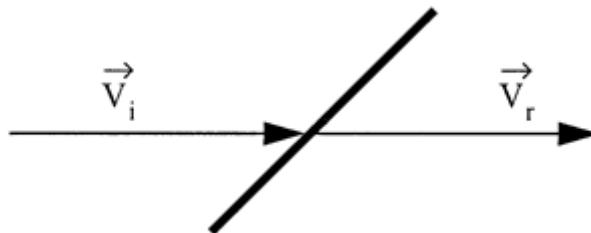
Figure 2.1-5 Direct transmission
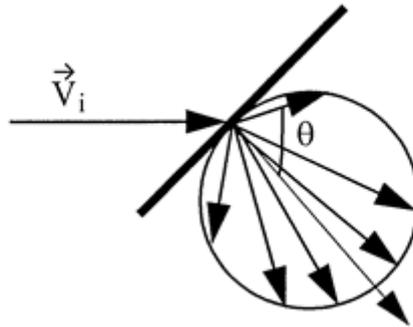
Figure 2.1-6 Diffuse transmission

When a <u>photon</u> (Solar Pressure only) strikes a surface its energy is distributed under three forms:

- **Absorption:** a fraction of the energy is absorbed by the surface.

- **Transmission:** for transparent or semi-transparent surfaces, a fraction of the energy passes through the surface. The transmission may be direct or diffuse.

- **Reflection:** the struck surface re-emits the energy which is not absorbed nor transmitted. The reflection may also be specular or diffuse.

The interaction particle/surface or photon/surface is described by the surface thermo-optical coefficients which are those used in thermal analyses. It is also possible to choose the "Ray emission" level (none, low, normal, critical) so to use more or less ray from this geometrical element.

Figure 2.1-7 Coating properties definition

All these coefficients may be defined as incident angle dependent parameters (see figure below). Note that the angle is defined in relation to the tangent to the shape, so that an angle of 0° represents a ray tangent to the surface, and an angle of 90° represents a ray normal to the surface.
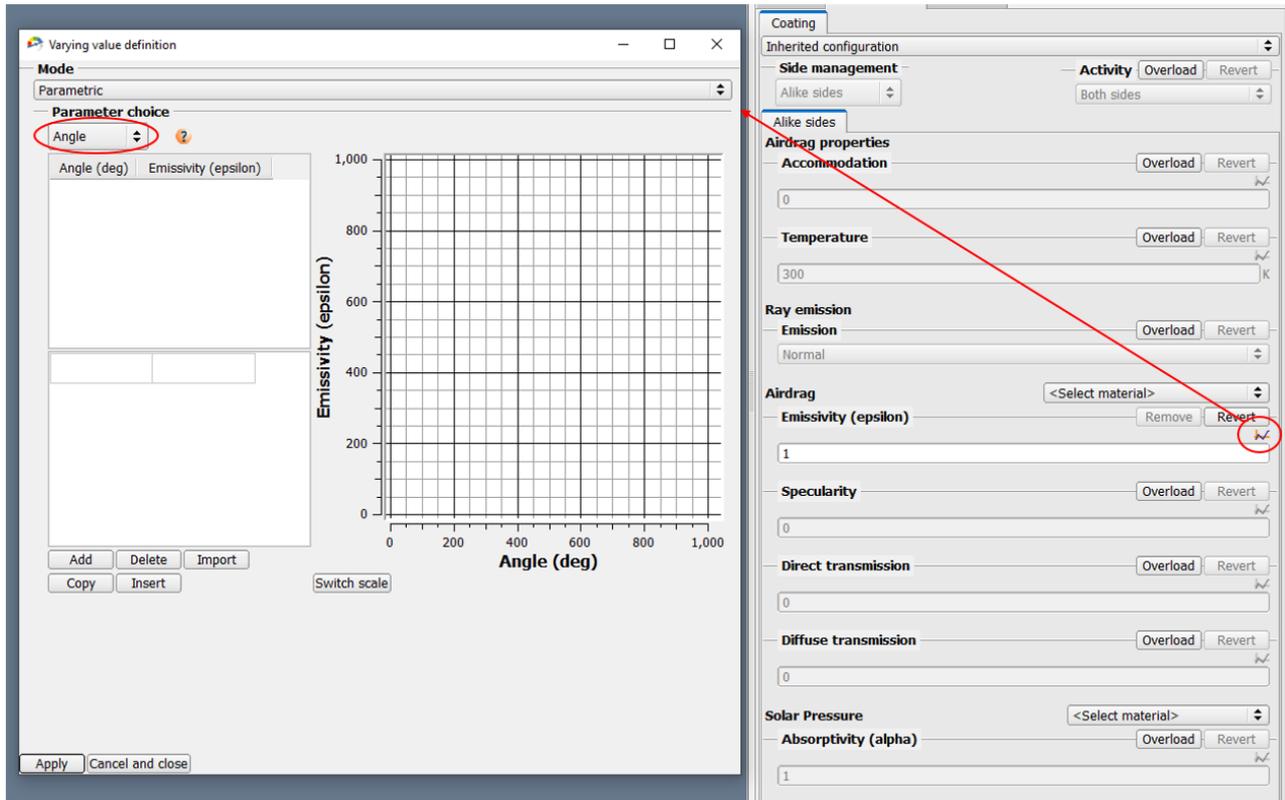
Figure 2.1-8 Incident angle dependent parameter definition

It is worth mentioning that the reflection and transmission coefficients are specified independently of the absorption coefficients, meaning that the energy/flux budget is split into the 4 modes

$$\rho_s + \rho_d + \tau_s + \tau_d = 1$$

so that the diffusive coefficients are in fact deduced from the other reflection/transmission modes:

$$\rho_d = 1 - \rho_s - \tau_s - \tau_d$$

The total energy/flux budget at a ray impact is then:

$$\alpha + (1 - \alpha)(\rho_s + \rho_d + \tau_s + \tau_d) = 1$$

It is also possible to assign a material to a geometrical element:
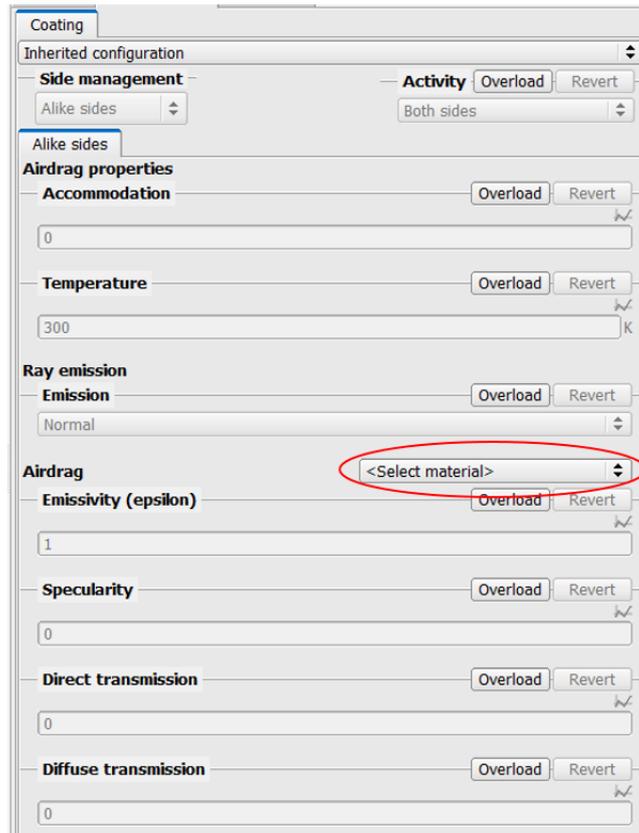
Figure 2.1-9 Material selection

In that case the material must be previously defined in the Material tab. The user can create a new material database or use the default one. Please refer to Systema User Manual (chapter "Geometrical model management") for more information about the use of Material tab. Once the material is created the material database to be used must be selected in the model tab by double clicking on the root level of the geometrical model.
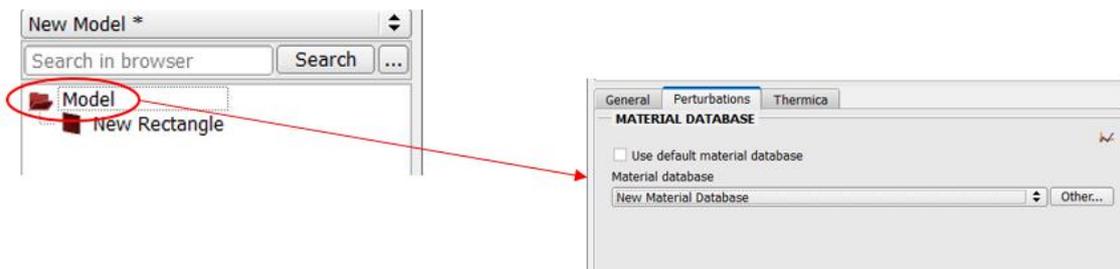
Figure 2.1-10 Material Database selection

The geometrical model must then be meshed. Creating a new meshing (File/New/meshing) will generate a basic meshing by default (one node per face). This meshing can then be modified and/or refined thanks to the meshing edition window. Please refer to the Systema User Manual (chapter "Geometrical model management") for more information about the meshing management.

2.2 Spacecraft on orbit description

Trajectory, Kinematics and Mission tabs of Systema are dedicated to the spacecraft on orbit description. They allow to define a trajectory and kinematic models that can then be associated with the geometric model in order to define the mission. The Mission tab is also used to define the computation points which correspond to the orbital positions at which the perturbing forces torques and fluxes will be computed by the software.

Please refer to the Systema User Manual for more information about the trajectory, kinematic and mission management.

2.3 Simulation definition

The definition of the simulation to be performed is done on the processing tab. This tab allows the user to create a diagram that will describe the different calculations to be performed. The Perturbations application contains two calculation modules that can be called or not in the simulation:

- Solar Pressure module: computes the solar radiation pressure caused by light fluxes incident on the spacecraft

- Air Drag: computes the perturbing forces, torques and thermal fluxes due to the atmosphere

Each module in the diagram can be edited to set the simulation parameters specific to that module.
The two modules and their simulation parameters are presented in detail in the next two chapters: "Solar Pressure module" and "Air Drag module". Please refer to the Systema User Manual (chapter "process management") for more information about the processing diagram management.

2.4 Forces and torques computation

The simulation can be launched thanks to the Run button ▶. During computation, the different menus and windows of screen are not accessible. Nevertheless, it is still possible to monitor the computation progress on the process run window that appears. The run is finished when the two progress bars indicate 100%.
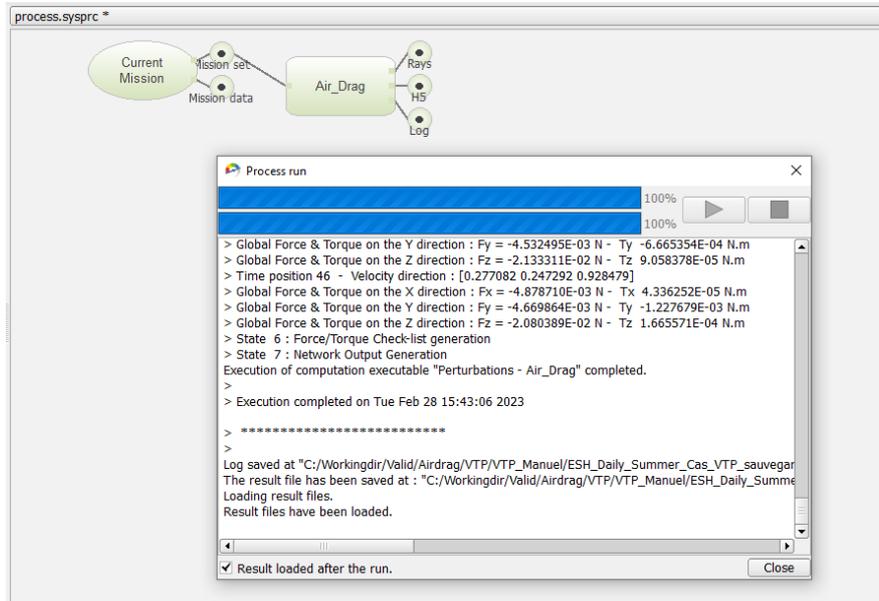
Figure 2.4-1 Process run window

2.5 Results analysis

At the end of the calculation, all results are stored in .h5 files in HDF5 format. Some additional information are also written in the .log files in txt format.

All the data in the .h5 can be displayed in the 2D or 3D views of Systema (in Modeler/Meshing and Mission tabs). They can also be post-processed in the post-processing tab.

In order to know how to display/post-process the results, please refer to the Systema User Manual (sections "Displaying the result of a process in tables and curves" or chapter "Post-processing management).

3 Solar Pressure module

3.1 Solar Pressure overview

The Solar Pressure module computes the solar radiation pressure which is the force produced by the impact of sunlight photons on the surface of the spacecraft. Radiation incident on the satellite's surfaces produces a radiation pressure, or force per unit area, equal to the difference between the incident and reflected momentum flux vector. The major factors determining the radiation pressure are:

- The intensity, the spectral and the spatial distribution of the incident radiation,
- The geometry of the satellite,
- The optical properties of the satellite surfaces.

We consider here the direct solar radiation only.

The Sun provides essentially collimated flux with a well-defined intensity and spectrum.

3.2 Solar Pressure parameters

3.2.1 Input

- .sysset file: Mission synthesis for the Perturbations application

3.2.2 Outputs

- **.sp.h5 file:** Contains force and torque results in HDF5 format to be displayed or post-processed into Systema.
- **.sp.log file:** log file
- **.sprt.h5 file (optional):** Contains all computed rays for display in Systema.

3.2.3 Parameters

The parameters are split into 2 general categories: Solar Pressure properties (local to the current module) and Common properties (global to the complete diagram).

Common properties

From the Common properties, the Solar Pressure module will be influenced by the "Sun Constant" category. It is possible to configure an automatic or manual Sun Constant.

Solar Pressure properties

Computation

- **Direct flux only:** If set to yes, it deactivates the multi-reflection and compute only the direct incident flux.

- **Sun modelling:** Choose between a finite or infinite Sun modelling. For a finite Sun, an additional Sun detail level is required corresponding to a refinement of the Sun meshing.
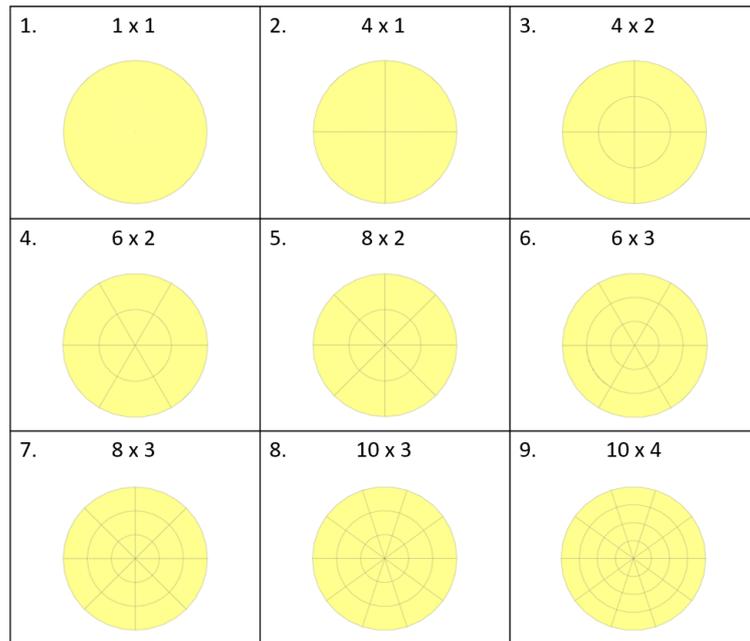
Figure 3.2-1 Detail levels of the Sun

The finite sun modelling should allow to take into account the shadow transition that is not considered with an infinite sun modelling.

- **Torques Reference Point:** X, Y and Z coordinates of the reference point used for the computation of torques.

Ray Tracing

- **Method:** Is used to choose between the Quasi Monte-Carlo or Monte-Carlo method for ray emission. The Quasi Monte-Carlo is the default method and is recommended for faster convergence.

- **Random seed:** Is used to modify the generation of pseudo random numbers used by the ray-tracing. This option may be modified to check the results sensitivity to the random process.

- **Ray threshold:** This option is used to filter the ray bounces according to the remaining energy of a ray. By default, this filtering is done whenever a ray has lost at least 99% of its energy (1% of remaining energy). In such cases, the remaining energy is considered as absorbed by the last impacted node.

Critical / Normal / Low

- **Ray building method:** Select "Per node" / "Per mesh" / "Density".

The default "Per node" value indicates that each node will use the given number of rays. In the case of condensed nodes (and/or with both active sides), this number of rays is distributed to the node's mesh according to their respective area (i.e. providing a uniform density of rays into the node). The "Per mesh" value may be used to specify the number of rays per radiative mesh (a both coating active mesh has in fact two radiative meshes: one per side). A condensed node will then use as many more rays as there are condensed radiative mesh into the node. The "Density" value allows to define a number of rays per square meter. The number of rays per mesh is still bounded between a minimum and maximum value defined by the user.

For computation performances, the total number of rays that shall be used for a node is re-distributed on the mesh so to keep a uniform density of rays.

This option shall be avoided when using CAD geometries because of those are built from a geometrical tessellation (and so with lots of mesh).

Outputs

- **Ray display length (reflection):** set the length of ray display to space. By default 2 (for reflection) and 2.5 (from Sun) times the size of the model.

3.3   Solar Pressure theoretical background

The present section deals with the radiation pressure originating from the Sun illumination. This force is the most important in geosynchronous orbit. Two parts are respectively dedicated to the modelling of the light energy received by the spacecraft, and to the modelling of the optical properties of the surfaces. The third part describes the computation scheme, with the ray-tracing process applied to the solar radiation pressure and the resulting forces and torques calculation.

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3.3.1 Radiation Flux

Due to the Sun's distance, the solar flux is considered as parallel to the direction Sun-spacecraft. The intensity of this flux is given by the solar constant E_0 (W/m2):

$$E_0 = \sigma T^4 \left(\frac{R_s}{D_s}\right)^2$$

where:

T: black body temperature, K

σ: Stefan-Boltzmann constant

R_s: sun radius (696 000 km in Systema)

D_s: distance Sun-planet (as a function of time)

Note that in Systema, the sun constant is calibrated to a value of 1371.3 W/m2 at a sun-planet distance equal to 149 597 870 km (which corresponds to T=5781.92 K).

3.3.2 Surface Optical Properties

When a photon strikes a surface its energy is distributed in three forms:

- Absorption: a fraction of the energy is absorbed by the surface.

- Transmission: for transparent or semi-transparent surfaces, a fraction of the energy passes through the surface. The transmission may be direct or diffuse. When direct, the photon keeps the same direction. When diffuse, the distribution function of the new direction follows the cosine law.

- Reflection: the struck surface re-emits the energy which is not absorbed or transmitted. The reflection may also be specular or diffuse. When specular, the normal component of the velocity direction is inverted whereas the tangential component is conserved.

According to these three phenomena the following coefficients define the surface optical properties:

- absorption coefficient: α

- transmission coefficients:

  o τ_s: for direct transmission

  o τ_d: for diffuse transmission

- reflection coefficients:

  o ρ_s: for specular reflection

  o ρ_d: for diffuse reflection

The optical coefficients are linked by the following relation:

$$\alpha + (1 - \alpha)(\rho_s + \rho_d + \tau_s + \tau_d) = 1$$

3.3.3 Computation Scheme

3.3.3.1 Ray-tracing

This paragraph deals with the trajectory of photons emitted towards the satellite. Reflection/transmission is also described.

The velocity direction of all the emitted particles is the Sun-planet direction. After impact with a surface the random process is the following:

A random number χ is selected from the interval [0,1],

- if $\chi \leq \rho_d$: diffuse reflection
- if $\rho_d < \chi \leq \rho_d + \rho_s$: specular reflection
- if $\rho_d + \rho_s < \chi \leq \rho_d + \rho_s + \tau_s$: direct transmission
- if $\rho_d + \rho_s + \tau_s < \chi$: diffuse transmission

When the energy of the photon reaches a minimum threshold, the ray-tracing process proceeds to the next one.

3.3.3.2 Resulting Force and Torque

In order to calculate force and torque generated by radiation pressure two options are available. The first only considers the first impact of photons on surfaces (direct flux), whereas the second accounts for multiple reflection (user selectable).

The momentum of the particle after impact has to be averaged. The elementary forces and torques are thus:

$$d\vec{F} = \frac{E_i}{c}\left((1 - (1-\alpha)(\rho_s + \tau_s))\vec{d_\iota} \; - \; 2(1-\alpha)\left(\rho_s cos\theta + \frac{\rho_d - \tau_d}{3}\right)\vec{n}\right)$$

$$d\vec{T} = \overrightarrow{OP} \wedge d\vec{F}$$

where:

$\vec{n}$	surface normal
$\vec{d_\iota}$	incident direction
θ	angle between surface normal and incident direction
E_i	energy carried by the emitted photon
c	speed of light
O	torque reference point
P	point of impact

E_i is given by:

$$E_i = \frac{E_0 \times S_1 \times \cos \theta_1}{N_1}$$

where:

S_1	area of the face from which the photon is emitted
θ_1	angle between face normal and sun direction
E_0	solar "constant" (W/m2)
N_1:	number of particles emitted from face 1

Note that the solar constant can be either automatically calculated or defined by the user (see the Common properties in the Solar Pressure parameters). If the automatic mode is selected, the solar constant value will be evaluated at each time and will depend on season.

4 Air Drag module

4.1 Air Drag overview

A satellite moving through the Earth upper atmosphere (or other planet atmosphere) is subjected to aerodynamic forces. The Air drag module computes perturbing forces, torques and thermal fluxes resulting from the interaction of the atmosphere particles with the satellite's surfaces. When a particle of atmosphere strikes a satellite's surface, it produces a force per unit area equal to the difference between the incident and reflected particle momentum flux vector. Therefore the air drag depends on the following parameters:

- the satellite velocity,

- the atmospheric constituents, their density and their kinetic temperature,

- the motion of the atmosphere,

- the geometry of the satellite,

- the orientation of the satellite w.r.t. the incoming flow,

- the surface properties and temperature.

This force is usually modelled by assuming that it acts in a direction opposite to the spacecraft's velocity vector relative to the ambient atmosphere. This direction can then be corrected by taking into account the wind velocity and the thermal velocity of particles.

The Air Drag module was initially developed to calculate the perturbing forces in Earth orbit. Calculations are based on the Earth atmosphere model NRLMSISE-00 associated to the HWM07 wind model. It is also possible to perform analysis for orbits around other planets but there is no detailed atmosphere model available. The user will be able to define a simplified atmosphere model with constant temperature, density and average molar mass values.

4.2 Air Drag parameters

4.2.1 Input

- .sysset file: Mission synthesis for the Perturbations application

4.2.2 Outputs

- **.adg.h5 file:** Contains force torque and thermal flux results in HDF5 format to be displayed or post-processed into Systema.

- **.adg.log file:** log file

- **.adrt.h5 file (optional):** Contains all computed rays for display in Systema.

4.2.3 Parameters

The parameters are split into 2 general categories: Air Drag properties (local to the current module) and Common properties (global to the complete diagram).

Common properties

The Air Drag module does not use any of these parameters.

Air Drag properties

Atmosphere parameters

- **Thermal Velocity:** If set to yes, the thermal velocity is taken into account to computes the velocity of the atmosphere particles and affects the ray direction. If not, the particles have all the same velocity which is equal to the opposite of the satellite velocity.

- **Atmosphere Model:** Choose between the Earth atmosphere model NRLMSIS00 (only for Earth orbit) or a user defined atmosphere model. The user defined model is much less accurate and therefore mostly useful for orbits around other planets than the Earth because there is no atmosphere model available. This model is based on constant Temperature, density and average molar mass values for the atmosphere. These values must be specified by the user.

- **Wind Velocity** (only for NRLMSIS00 atmosphere model)**:** If set to yes, the wind velocity is taken into account to computes the velocity of the atmosphere particles and affects the ray direction. HWM07 is the only wind model available. If set to no, the particles have all the same velocity which is equal to the opposite of the satellite velocity (plus thermal velocity if enabled).

- **Ap** (only for NRLMSIS00 atmosphere model)**:** Earth magnetic index, which is an input for the NRL-MSIS00 and the HWM07 models.

- **F10.7** (only for NRLMSIS00 atmosphere model)**:** Daily F10.7 solar flux (10-22 W/m2/Hz), which is an input for the NRL-MSIS00 and the HWM07 models.

Computation

- **Computation method:** Choose the method used to computes the perturbing forces, torques and fluxes. Two methods are available:

  o The "Exact" method is based on the ray tracing technique to compute the perturbing effects at the ray level. A force, torque and flux are calculated at each ray impact. They are then summed to obtain one value per node. This method takes into account multi-reflections of rays but often need a larger number or rays than the analytical method to converge.

  o The Analytical method is based on integrated equations and the perturbing effects are directly computed at node level. The ray tracing technique is only used to handle masking effects. This method does not take into account multi-reflections of rays but often need a smaller number or rays than the exact method to converge.

- **Direct flux only:** If set to yes, it deactivates the multi-reflection and compute only the direct incident flux.

- **Torques Reference Point:** X, Y and Z coordinates of the reference point used for the computation of torques.

Ray Tracing

- **Method:** Is used to choose between the Quasi Monte-Carlo or Monte-Carlo method for ray emission. The Quasi Monte-Carlo is the default method and is recommended for faster convergence.

- **Random seed:** Is used to modify the generation of pseudo random numbers used by the ray-tracing. This option may be modified to check the results sensitivity/ to the random process.

- **Ray threshold:** This option is used to filter the ray bounces according to the remaining energy of a ray. By default, this filtering is done whenever a ray has lost at least 99% of its energy (1% of remaining energy). In such cases, the remaining energy is considered as absorbed by the last impacted node.

Critical / Normal / Low

- **Ray building method:** Select "Per node" / "Per mesh" / "Density".

The default "Per node" value indicates that each node will use the given number of rays. In the case of condensed nodes (and/or with both active sides), this number of rays is distributed to the node's mesh according to their respective area (i.e. providing a uniform density of rays into the node). The "Per mesh" value may be used to specify the number of rays per radiative mesh (a both coating active mesh has in fact two radiative meshes: one per side). A condensed node will then use as many more rays as there are condensed radiative mesh into the node. The "Density" value allows to define a number of rays per square meter. The number of rays per mesh is still bounded between a minimum and maximum value defined by the user.

For computation performances, the total number of rays that shall be used for a node is re-distributed on the mesh so to keep a uniform density of rays.

This option shall be avoided when using CAD geometries because of those are built from a geometrical tessellation (and so with lots of mesh).

4.3 Air Drag theoretical background

The present section deals with the modelling of the (neutral) air drag under free-molecular flow conditions. In the first paragraph the modelling of the ambient atmosphere is presented. The second paragraph deals with the molecular flux. Then interaction of the atmosphere particles with the spacecraft surfaces is studied. Finally the computation scheme of the Air Drag module is presented.

4.3.1 Ambient atmosphere

The free-molecular flow conditions are described hereafter:

Let l define the mean free molecular path length, which is the average distance for a molecule between two collisions. A formula for the mean free path can be obtained if molecules are modelled as "hard spheres", with their collisions being like those of billiard balls (Maxwell):

$$\lambda = \frac{l}{\sqrt{2}n\sigma^2}$$

where:

n   the number density, 1/m3

σ the collision cross section between two molecules, m2

If L is a typical length of the spacecraft, the free-molecular flow is assumed when the Knudsen number Kn conforms with:

$$Kn = \frac{\lambda}{L} \gg 10$$

In that case, the molecules suffer very few collisions in the region of interest and can be considered independent from one another.

4.3.1.1 Molecule Characteristics

The parameters of the atmosphere required for the air drag calculation are:

- n: number density of particles (part./m3)
- T: temperature of the atmosphere (Kinetic, K)
- M: average molar mass (kg/kmol)

These parameters depend on the following geometrical factors:

- z: satellite geodetic altitude
- φ: satellite geodetic latitude
- λ: satellite longitude (+ towards east)
- h: satellite local solar time
- δ: solar declination
- Λ: solar celestial longitude (+toward east)

and on physical parameters:

- $F_{10.7}$: solar flux. This quantity is assumed to be well known in the 10.7 centimeter radio band
- A_p: linear geomagnetic activity index

In order to solve the problem rigorously, the overall theory should include:

- the integration of dynamic (Navier-Stokes) equations concerning the whole atmospheric fluid,

- the knowledge of boundary conditions, that is the atmospheric conditions at the limits of the atmosphere

- the knowledge of energy exchange mechanisms in the whole atmosphere.

It is easy to understand that assumptions are made to simplify the problem:

- the boundary conditions are known,

- the vertical columns of the atmosphere are independent from one another, and are in a static equilibrium of diffusion,

- each constituent of the atmosphere can be considered separately.

4.3.1.2 The NRLMSISE-00 atmosphere Model

The NRL Mass Spectrometer, Incoherent Scatter Radar Extended Model (NRLMSISE-00) model describes the neutral temperature and species densities in Earth's atmosphere. It is based on a very large underlying set of supporting data from satellites, rockets and radars, with extensive temporal and spatial distribution. It has been extensively tested by the global scientific community against experimental data. The model has a flexible mathematical formulation. It is valid for use from ground level to the exosphere. The Mass Spectrometer and Incoherent Scatter (MSIS) series of models developed between 1977 and 1990 are used extensively by the scientific community for their superior description of neutral composition. The models utilized atmospheric composition data from instrumented satellites and temperatures from ground-based radars. The initial MSIS 1977 model was based on the Jacchia temperature profile framework, but the density at 120 km varied with local time and other geophysical parameters to fit the measurements. Exospheric temperature and density variations were represented by spherical harmonics resulting in requiring fewer parameters for a given level of accuracy. Subsequent versions of the model include the longitude variations, a refined geomagnetic storm effect, improved high latitude, high solar flux data and a boundary lowered to sea level. The NRLMSISE-00 model of atmospheric composition, temperature, and total mass density from ground to exobase includes the following:

- Drag data based on orbit determination

- More recent accelerometer data sets

- New temperature data derived from Millstone Hill and Arecibo incoherent scatter radar observations

- Observations of O2 by the Solar Maximum Mission (SMM), based on solar ultraviolet occultation.

For this model, the following parameters are used.

Input parameters:

Year and day in the year (YYDDD)

Seconds in the day (UT seconds)

Altitude (kilometres)

Latitude (degrees)

Longitude (degrees)

Local solar time (decimal hours)

Daily F10.7 solar flux (10-22 W/m2/Hz)

3 month average F10.7 solar flux (10-22 W/m2/Hz)

Magnetic index (Ap)

Mass number of the selected species

Output parameters:

Helium number density (/cm3)

Atomic oxygen number density (/cm3)

Molecular nitrogen number density (/cm3)

Molecular oxygen number density (/cm3)

Argon number density (/cm3)

Atomic hydrogen number density (/cm3)

Atomic nitrogen number density (/cm3)

Total density (g/cm3)

Exospheric temperature (K)

Temperature at altitude (K)

4.3.2 Molecular Flux

In order to determine the molecular flux (part/m2/s), the velocity of particles has first to be calculated. Then, knowing the density of particles and orientation of a surface element the flux is given for any surface.

4.3.2.1 Particle velocity

The velocity of a particle is the sum of a group velocity (aerodynamic velocity) and an individual velocity (thermal motion). For the computation of the thermal motion the Maxwellian theory of gas dynamics is used. In particular, this theory assumes an isotropic distribution.

The velocity $\vec{v}$ is given by:

$$\vec{v} = \vec{v}_{aero} + \vec{u}$$

where:

$\vec{u}$	thermal motion vector
$\vec{v}_{aero}$	aerodynamic velocity vector

The distribution function of the thermal motion vector is:

$$f(\vec{u}) = \frac{1}{\pi^{3/2} u_m^3} \exp\left(-\frac{u^2}{u_m^2}\right) \tag{4-1}$$

but it is the same to say that the direction is random in the 4π steradians, whereas the thermal motion magnitude is given by the following distribution function:

$$f(u_i) = \frac{1}{\sqrt{\pi} u_m} \exp\left(-\frac{u_i^2}{u_m^2}\right)$$

where:

u_m	thermal velocity
u_i	x,y or z coordinate of thermal motion.

u_m is defined by:

$$u_m = \sqrt{\frac{2RT}{M}}$$

where:

T	ambient temperature (K)
R	universal gas constant, $R = 8314 \, \text{J/kmol/K}$
M	molecular weight (kg/kmol)

The aerodynamic velocity is the difference between the wind velocity and the spacecraft velocity:

$$\vec{v}_{aero} = \vec{v}_w - \vec{v}_{s/c}$$

where:

$\vec{v}_w$	wind velocity in inertial frame determined using a wind model
$\vec{v}_{s/c}$	spacecraft velocity in inertial frame

4.3.2.2 Wind Model

The wind model used in Systema-Perturbations is called HWM07. It is an empirical model of horizontal winds in the troposphere, stratosphere, mesosphere, and thermosphere, and is intended to succeed HWM93 [Hedin et al., J. Atmos. Terr. Phys., vol. 58, 1421-1447, 1996]. In addition to the data used in HWM93, the model is based on extensive new ground-based and space-based wind measurements, including height profiles from NASA-UARS/WINDII, NASA-UARS/HRDI, measurements from ground-based optical and radar instruments obtained from the NSF-CEDAR database, and lower atmospheric NCEP data.

In the thermosphere, the model consists of two parts: a quiet-time portion, and a geomagnetically disturbed portion. The quiet-time part represents average wind conditions when ap <= 12. The disturbed part represents average perturbation winds for the specified ap input.

The quiet part is represented by vector spherical harmonics in geodetic latitude, geodetic longitude, and solar local time, up to wave number 8 in latitude, 2 in longitude, and 3 in local time. The seasonal dependence is represented by harmonic terms up to semiannual. The vertical structure is represented below 250 km by cubic B-splines with node spacing of 5 km below 110 km and higher nodes at 110, 117, 125, 135, 150, 200, and 250 km. Above 250 km, an exponential decay function with a scale height of 60 km is used; continuity up to the second derivative is imposed at 250 km.

The disturbance winds depend on magnetic latitude, magnetic local time, and Kp. The Quasi-Dipole magnetic coordinates described by Richmond [J Geomagn. Geoelectr., vol. 47, 191-212, 1995] are used for the magnetic coordinates; the code was obtained from the NSF-CEDAR database, and the interpolation grid was computed from IGRF at 250 km and epoch 1994.0 (a modified FORTRAN-90 version of the code, apexcord.f90, is included in this package; it contains only those subroutines needed for reading the interpolation grid and computing QD coordinates). The magnetic latitude and magnetic local time dependence of the disturbance winds is represented by vector spherical harmonics up to wave number 10 in magnetic latitude and wave number 3 in magnetic local time. At mid and low latitudes, only latitudinal terms up to wave number 4 are used; the transition from low resolution at low latitudes to high resolution at high latitudes occurs at a pre-determined latitude that depends on local time and Kp. The transition is made with an exponential function with a width of 4 degrees. The Kp dependence is represented by cubic splines with nodes at 0, 2, 5, and 8. The Kp dependence is constrained to have zero slope at Kp=0 and Kp=8, and is constant above Kp=8.

Input parameters :

> Year and day (YYDDD)
> Seconds in the day (UT seconds)
> Altitude (kilometres) (range : 100 – 850 km)
> Latitude (degrees)
> Longitude (degrees)
> Local solar time (decimal hours)
> Daily F10.7 solar flux (10-22 W/m2/Hz)
> 3 month average F10.7 solar flux (10-22 W/m2/Hz)
> Magnetic index (Ap)

Output parameters :

> Meridional wind (northward) (m/s)
> Zonal wind (eastward) (m/s)

4.3.2.3 **Flux**

Let Φ be defined as the flux density of particles on a surface, per time unit and square meter. Assuming that the inward surface normal gives the direction of the axis x, one has to calculate the average velocity on x of particles having $v_x > 0$.
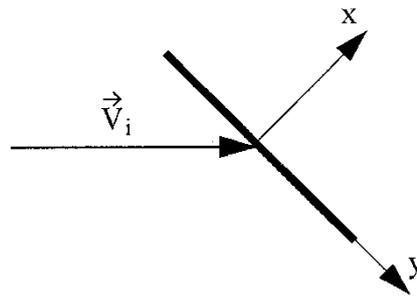
This average velocity is given above:

$$\overline{v_x} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{0}^{\infty} v_x \ f(\vec{v} - \vec{v}_{aero}) dv_x \ dv_y \ dv_z$$

Completing the integration:

$$\Phi = n \sqrt{\frac{RT}{2\pi M}} \left(\exp(-s^2) + \sqrt{\pi} s (1 + \text{erf}(s)) \right)$$

where:

n mean number density of the atmosphere, part./m3
s molecular speed ratio
erf error function

s is given by:

$$s = \frac{\vec{v}_{aero} \cdot \vec{x}}{u_m}$$

$erf(s)$ is defined by:

$$\text{erf}(s) = \frac{2}{\sqrt{\pi}} \int_{0}^{s} \exp(-x^2) \, dx$$

which can be approximated with the following asymptotic series:

s>0 $\qquad \mathrm{erf}(s) = 1 - \frac{1}{\sqrt{\pi}} \exp(-s^2) \left(\frac{1}{s} - \frac{1}{2s^3} + \frac{1}{2^2 s^5} - \cdots \right)$

s<0 $\qquad erf(s) = -erf(-s)$

4.3.3 Particle/Wall interaction

4.3.3.1 Problem presentation

The impingement of molecules on surfaces is a physical phenomenon difficult to model since, up to now, few experimental results exist on the subject. The main features to be considered in the reflection model are the following:

- the re-emission direction depends on the angle of impingement,
- direction and velocity of the reflected particle depend on the surface material.

A study of reflection models has been performed by P.Duque (DUQUE, 1987), each of them trying to integrate the above mentioned characteristics. For skin force analysis, the model used will be called modified Maxwellian model since it is an extension of the Maxwellian model described in (DUQUE, 1987).

4.3.3.2 Modified Maxwellian Model

This model accounts for specular and diffuse reflection, and also for absorption and transmission. Adding absorption and transmission enables to consider:

- for absorption: deposits of particles on sensitive surfaces,
- for transmission: grid surfaces (trusses, antennas).

Three coefficients are defined in order to give the distribution between reflection, absorption and transmission:

- α, absorption coefficient, is the probability for a particle to be absorbed,
- τ, transmission coefficient, is the probability for a particle to be transmitted,
- ρ, reflection coefficient, is the probability for a particle to be reflected.

These coefficients are related by:

$$\alpha + (1 - \alpha)(\rho + \tau) = 1$$

4.3.3.3 Specular reflection

The incoming particle is reflected mirror-like, that is, the normal component of velocity w.r.t. the surface is reversed, and the tangential component is conserved.
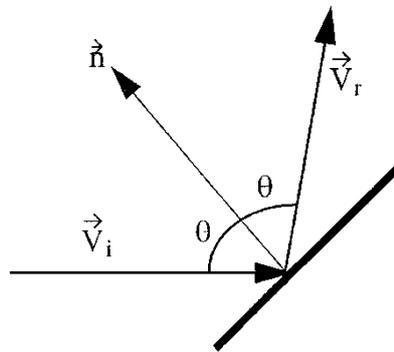
The reflected velocity vector is given by:

$$\vec{v}_r = \vec{v}_i - 2(\vec{v}_i.\vec{n})\vec{n}$$

where:

$\vec{v}_i$ incident velocity vector
$\vec{n}$ surface normal
$\vec{v}_r$ reflected velocity vector

4.3.3.4 Diffuse reflection

Diffuse reflection means that the reflection direction is random and is not correlated with the incident direction. Two cases can occur:

- the surface microroughness disperses the particle. In that case the velocity magnitude is kept.
- the particle enters the surface structure, then is re-emitted with agitation temperature equal to the surface temperature.

In both cases, the re-emission flux follows the cosine law. This law assumes that the direction distribution function is proportional to the cosine of the angle between re-emission direction and surface normal.
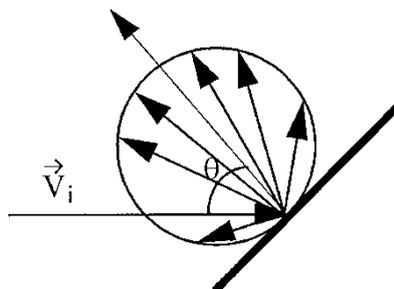
When the particle is re-emitted at wall temperature, the velocity magnitude distribution function is deduced from the velocity distribution function as explained in (BIRD, 1976):

$$f(v_r) = \frac{4\pi}{u_s^3} v_r^2 \exp\left(-\frac{v_r^2}{u_s^2}\right)$$

where:

v_r velocity magnitude of reflected particle

u_s thermal velocity at surface temperature

u_s is given by:

$$u_s = \sqrt{\frac{2RT_s}{M}}$$

where:

T_s surface temperature

In that case, the velocity is small w.r.t. to the incident aerodynamic velocity. It means that the reflected particle will have a weak influence on the air drag computation. Therefore we assume that the reflected particle velocity is equal to the mean velocity magnitude which is given by the theory:

$$\bar{u} = \sqrt{\frac{\pi}{2} \frac{RT_s}{M}}$$

For symmetry reasons, the average direction is the surface normal. Therefore the mean reflected velocity vector is:

$$\vec{v}_r = \sqrt{\frac{\pi}{2} \frac{RT_s}{M}} \vec{n}$$

In order to weigh these three types of reflection, three coefficients are defined from the domain [0,1]:

- ρ_s, coefficient of specular reflection, is the probability for a re-emitted particle to be reflected specularly
- ρ_d, coefficient of diffuse reflection, is the probability for a re-emitted particle to be reflected diffusely
- σ, accommodation coefficient, is the probability for a particle diffusely re-emitted to be re-emitted at surface temperature.

The probability for an incident particle to be reflected specularly is $(1 - \alpha)\rho_s$.
The probability for an incident particle to be reflected diffusely is $(1 - \alpha)\rho_d$.
The probability to have a diffuse reflection at surface temperature is thus $(1 - \alpha)\rho_d\sigma$.

One can notice that the specular reflection permits to consider the dependence on the incidence angle, while the diffuse reflection accounts for surface effects. The temporal evolution of surface properties is not considered, but could be modelled by diffusion and accommodation coefficients varying with time.

4.3.3.5 Direct transmission

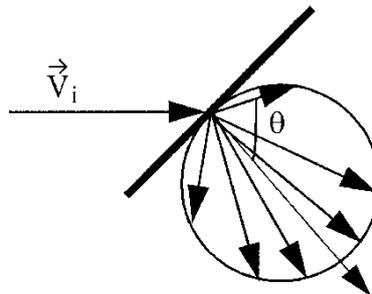The particle passes through the surface without deviation.
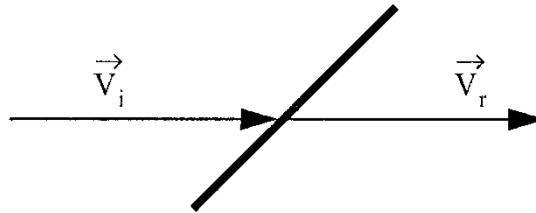
The transmitted velocity vector is given by

$$\vec{v}_r = \vec{v}_i$$

4.3.3.6 Diffuse transmission

The transmission direction is random and follows the cosine law.

As for the diffuse reflection, the particle can be re-emitted at the same velocity magnitude or at surface temperature depending on the accommodation coefficient.

In order to weigh these three types of transmission, three coefficients are defined from the domain [0,1]:
- τ_s, coefficient of direct transmission, is the probability for a re-emitted particle to be transmitted specularly
- τ_d, coefficient of diffuse reflection, is the probability for a re-emitted particle to be transmitted diffusely
- σ, accommodation coefficient, is the probability for a particle diffusely re-emitted to be re-emitted at surface temperature.

The probability for an incident particle to be transmitted specularly is $(1 - \alpha)\tau_s$.
The probability for an incident particle to be transmitted diffusely is $(1 - \alpha)\tau_d$.
The probability to have a diffuse transmission at surface temperature is thus $(1 - \alpha)\tau_d\sigma$.

4.3.4 Algorithm

This section details the main steps of the Air Drag algorithm for Exact and Analytical methods.

4.3.4.1 Exact method

The Exact method is based on the ray tracing technique to compute the perturbing effects at the ray level. Force, torque and flux are calculated at each ray impact. They are then summed to obtain one value per node. This method takes into account multi-reflections of rays but often need a larger number or rays than the analytical method to converge.

First, the ray-tracing is initialized at the beginning of each time step and the following quantities are calculated:
- The aerodynamic velocity of the particles which is a combination of the spacecraft velocity and the wind velocity (if enabled)
- The density ρ, the temperature T and the average molar mass M of the atmosphere

The total number of particles to be emitted is specified by the user. The ray-tracing algorithm is applied for each particle:

- **Random emission point**
  Emission points are sampled randomly, according to an iso-distribution law, with an analytic definition of shapes.

- **Random emission velocity vector**
  Each point is tested to know whether it is oriented towards the satellite velocity. Then, ray-tracing is used to test whether the point is occulted or not (i.e in the shade of another surface or not). To do so, an emission direction has to be determined for each particle. If the thermal velocity option is OFF, the emission direction is the opposite of the aerodynamic velocity vector. If the thermal velocity option is ON, a random velocity vector has to be determined. Three numbers, χ_x, χ_y and χ_z, are selected at random in the [0,1] interval such that:

$$\chi_x = \int_{-\infty}^{u_x} f(u)\,du \qquad \chi_y = \int_{-\infty}^{u_y} f(u)\,du \qquad \chi_z = \int_{-\infty}^{u_z} f(u)\,du$$

Where f is the distribution function of the thermal motion vector given by equation $f(\vec{u}) = \frac{1}{\pi^{3/2} u_m^3} \exp\left(-\frac{u^2}{u_m^2}\right)$ (4-1).

After integration:

$$\chi_x = \frac{\mathrm{erf}\left(\frac{u_x}{u_m}\right) + 1}{2} \qquad \chi_y = \frac{\mathrm{erf}\left(\frac{u_y}{u_m}\right) + 1}{2} \qquad \chi_z = \frac{\mathrm{erf}\left(\frac{u_z}{u_m}\right) + 1}{2}$$

These equations are inverted, to give the coordinates of the thermal motion vector:

$$u_x = u_m \operatorname{erf}^{-1}(2\chi_x - 1)$$
$$u_y = u_m \operatorname{erf}^{-1}(2\chi_y - 1)$$
$$u_z = u_m \operatorname{erf}^{-1}(2\chi_z - 1)$$

- **Re-emission of particles after impact**

  The re-emission random process is the following:
  - A random number χ is taken from [0,1],
  - if $\chi \le \sigma\rho_d$:   diffuse reflection at surface temperature
  - if $\sigma\rho_d < \chi \le \rho_d$:   diffuse reflection with same velocity magnitude
  - if $\rho_d < \chi \le \rho_d + \rho_s$:   specular reflection
  - if $\rho_d + \rho_s < \chi \le \rho_d + \rho_s + \tau_s$:   direct transmission
  - if $\rho_d + \rho_s + \tau_s < \chi \le \rho_d + \rho_s + \tau_s + \sigma\tau_d$: diffuse transmission at surface temperature
  - if $\rho_d + \rho_s + \tau_s + \sigma\tau_d < \chi$:   diffuse transmission with same velocity magnitude

  The re-emission direction and the velocity are determined as described in 4.3.3 Particle/Wall interaction according to the type of re-emission.

- **Force, torque and thermal flux computation**

$$\vec{F} = \frac{N_{impact}}{N_{emit}} \rho v_i \cos\theta\, S\, (\vec{v}_i - (1-\alpha)\vec{v}_r)$$

$$\overrightarrow{OP} \wedge \vec{F}$$

$$\Phi_{th} = \frac{N_{impact}}{N_{emit}} (\phi_i - (1-\alpha)\phi_r)$$

where

$\phi_i = \frac{1}{2}\rho v_i^3 \cos\theta S$

$\phi_r = \rho v_i \cos\theta S \times \frac{2RT_s}{M}$ for a diffuse reflection or transmission at surface temperature

$\phi_r = \phi_i$ otherwise

- **Propagation of rays after first impact and search for new impact**

  The ray has started from the initial emitting node with a particle flux N_0. During its travel the ray transfers the flux to each impacted node. The ray is re-emitted from the impacted surface if the flux is greater than the extinction threshold ("Ray threshold" parameter in the Airdrag module of processing tab). If not, the ray-tracing processes the next ray.

  If the ray is re-emitted, the main problem is to determine which surface is struck by the ray. An optimized search is needed as it is impossible to test all the surfaces. The ray-tracing considers the analytic definition of shapes.
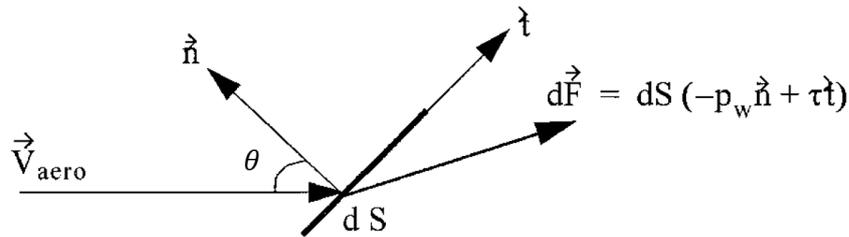
4.3.4.2 Analytical method

The Analytical method is based on integrated equations and the perturbing effects are directly computed at node level. This method does not take into account multi-reflections of rays but often need a smaller number

or rays than the exact method to converge.

The algorithm is similar to the one used for the Exact method except that the ray-tracing is only used to test whether the point is occulted or not, so to handle masking effects. Then the forces, torques and thermal fluxes are computed at node level:

$$d\vec{F} = \left(-P_w \vec{n} + \tau \vec{t}\right)dS$$

$$d\vec{T} = \overrightarrow{OP} \wedge d\vec{F}$$

$$\Phi_{th} = \alpha \Phi_i + (1-\alpha)\sigma(\rho_d + \tau_d)(\Phi_i - \Phi_r)$$

Where

$\vec{n}$ is the surface normal vector

$\vec{t}$ is the surface tangent vector

P_w is the normal pressure

τ is the stress pressure

O   is the Torque reference point

P   is the emission point

$$P_w = P_\infty \left[\left(1 + (1-\alpha)(\rho_s + \tau_s)\right)\frac{1}{\sqrt{\pi}}\Pi(s\cos\theta) + \frac{\sigma(1-\alpha)(\rho_d - \tau_d)}{2}\sqrt{\frac{T_s}{T}}X(s\cos\theta)\right.$$
$$\left. + \frac{2}{3}(1-\sigma)(1-\alpha)(\rho_d - \tau_d)\frac{1}{\sqrt{\pi}}\Pi(s\cos\theta)\right]$$

$$\tau = P_\infty(1 - \rho_s - \tau_s)\frac{1}{\sqrt{\pi}}X(s\cos\theta)\, s\sin\theta$$

$$\Phi_i = \frac{\rho RT}{M} \times \sqrt{\frac{RT}{2M}} \times \left[\frac{1}{\sqrt{\pi}}(s^2 + 2)X(s\cos\theta) + \frac{1}{2}s\cos\theta\,(1 + \mathrm{erf}(s\cos\theta))\right]$$

$$\Phi_r = \frac{\rho RT}{M} \times \sqrt{\frac{RT}{2M}} \times \frac{2}{\sqrt{\pi}}\frac{T_s}{T}X(s\cos\theta)$$

where

$$P_\infty = \frac{\rho R T}{M}$$

$$s = \frac{v_{aero}}{\sqrt{2RT/M}}$$

$$X(x) = \exp(-x^2) + x\sqrt{\pi}(1 + \mathrm{erf}(x))$$

$$\Pi(x) = xX(x) + \frac{\sqrt{\pi}}{2}(1 + \mathrm{erf}(x))$$

θ is the angle between aerodynamic velocity vector and surface normal vector

ρ is the atmosphere density (kg/m3)

M is the molecular weight (kg/kmol)

T is the ambient temperature

T_s is the surface temperature

Note that the ambient air flux is considered as a parallel flux in the Analytical method (i.e the velocity is the aerodynamic velocity vector) because the Maxwellian component of the velocity (thermal motion vector) has been directly "integrated" in the equations.

There is no propagation of rays after first impact because only first impact is considered with the analytical method.

5 Bibliography

BIRD, G. (1976). *Molecular gas dynamics.*

DUQUE, P. (1987, December). Aerodynamic Forces and Moments of Free-Molecular Flow. ESA. *(Working paper 347).*